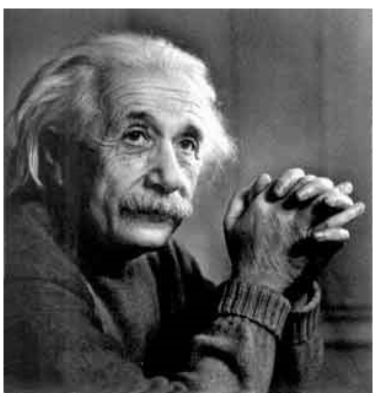
Chapter 12: Quantum Strangeness

In the television show “The Big Bang Theory”, Dr. Sheldon Cooper describes the best use of his time as a scientist to “*employ his rare and precious mental faculties to tear the mask off of nature and stare at the face of God.*” [1] And while the fictitious character may have an inflated view of the magnitude of his research efforts, he is not in poor company in terms of the feelings that science is a tool to be used to see the nature of God in nature itself. Albert Einstein is quoted as saying “*Science without religion is lame. Religion without science is blind.*” [2] Another quote attributed to Einstein is, “*I want to know God’s thoughts; the rest is just details.*” [3] Of course, Einstein claimed some familiarity with the intentions of the Creator when he quipped in a letter to Max Born, “*Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the "old one." I, at any rate, am convinced that He does not throw dice.*” [4]



Much has been made of Einstein’s opinions of God as a craps player. Through the 1920s and 1930s, Einstein and Niels Bohr had many conversations on the ramifications of the quantum theory. In response to Einstein’s quip about a non-dice-playing deity, Bohr is said to have responded, “*Einstein, stop telling God what to do!*”[1] Of course, Bohr was very well aware of the strangeness of the quantum theory and how it shook the very roots of conventional wisdom about nature. Bohr is quoted as saying, “*Anyone who is not shocked by quantum theory has not understood it.*” [5]

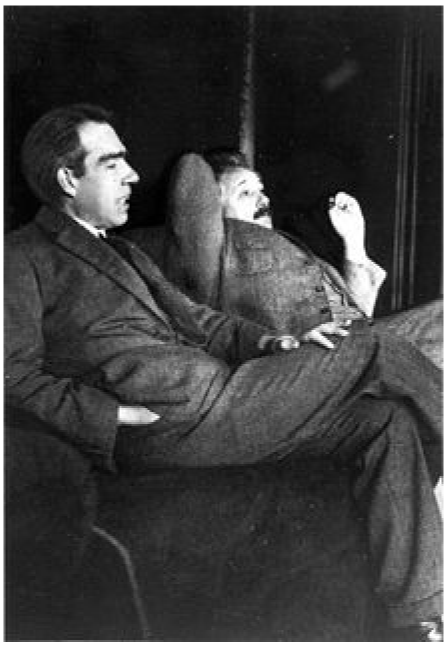
Naturally, Einstein found quantum theory quite shocking indeed. One of his earliest objections was that the quantum theory required that one dismiss a deterministic view of the universe. The philosophy of **Determinism** states that if all is known about a system at one point in time, then all can be known about that system at all points in time. Bohr, on the other hand, had no difficulties in dismissing determinism in favor of a quantum theory. Eventually, the debate would focus on the indeterminacy predicted by the Heisenberg Uncertainty Principle for complimentary variables (variables for which the corresponding quantum mechanical operators do not commute, such as position and momentum.)

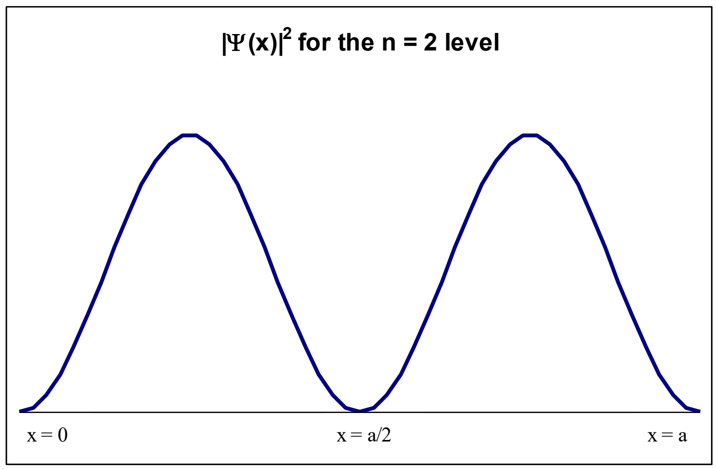
In fact, the spirited (but mostly amiable) debates between Einstein and Bohr did the development of quantum theory an enormous service. (not all of Bohr’s debates were amiable. Some of his discussions with Werner Heisenberg left Heisenberg reportedly in tears! Heisenberg

[1] This quote, while very clever, is disputed, as a very similar quote is also attributed to Enrico Fermi.

said of these discussions, "*Since my talks with Bohr often continued till long after midnight and did not produce a satisfactory conclusion, ...both of us became utterly exhausted and rather tense.*") [6]

By poking at the forefronts of what the theory predicts and what it can not predict, the Bohr-Einstein debates pushed quantum theory forward by enormous leaps. In this chapter, we will examine how various people have probed the "strangeness" of the quantum theory and the bizarre behavior it predicts (or in some cases, the bizarre behavior that was discovered almost by accident.) Much of the strangeness of quantum mechanics continues to be researched actively and colors such important topics as quantum communications and quantum computing.





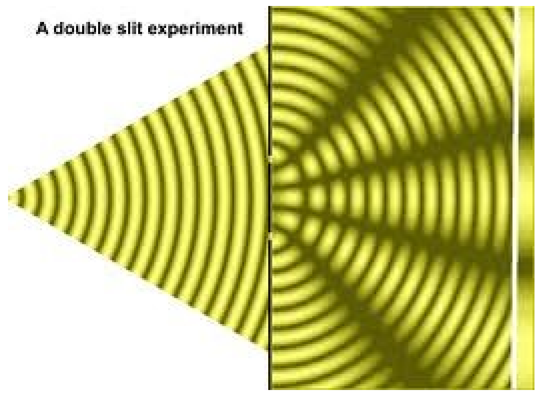
Nodes and Wave Nature

One of the first introductions students of the Quantum Theory receive involves the nodes in the wavefunctions of a one-dimensional particle in a box. The probability of measuring the particle to exist at any given position in the box is given by the square of the wavefunction. For the n = 2 level, the squared wavefunction is plotted above.

The figure shows that the probability of measuring the position of the particle at positions x = a/4 and x = 3a/4 or the maxima and that there is zero probability of measuring the particle to exist at the endpoints or at x = a/2, the middle position of the box. One might wonder how the particle can travel from one side of the box to the other without ever actually being in the middle. If one models the particle as a small ball bearing traveling from end to end in an evacuated, sealed glass tube (consistent with a deterministic view in which the particle has a definite location at all times) the prediction is clearly troubling. For many, this creates a dilemma.

The reconciliation of this dilemma requires that one abandon a notion of determinism in embracing the wave-nature of the particle. Namely, if one accepts the wave description of the particle, the notion of a definite location become meaningless since the wave must be delocalized across the entire box. In fact, the wave even exists at the central node despite the value of the wavefunction being zero! This concept provides a clear challenge to the notion of determinism that is suggested by Newtonian physics. The idea of “matter waves” also lead to a proposal by Louis de Broglie that matter-wave interference should be observable.

Quantum Interference

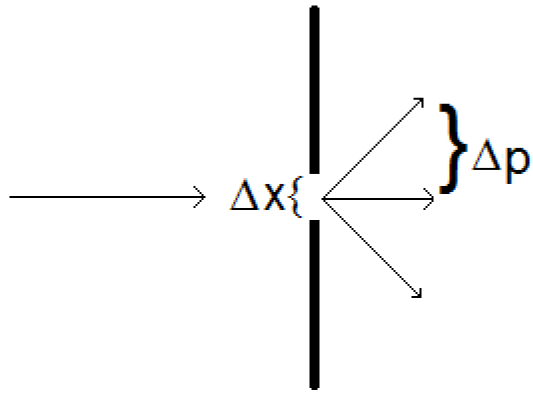


Thomas Young showed in 1803 [7] that light traveling through a pair of parallel slits will produce an interference pattern that follows Bragg’s Law for diffraction. This was a huge problem to the existing Newtonian theory of light, as Newton had postulated that light is, in fact, a stream of particles. With the advent of a quantum theory, light was postulated to have a dual nature, having properties of both particles and waves. This dual nature, of course would be applicable to the description of matter as well according to Louis de Broglie. At this point, things started to get really interesting. But before we go into that, let’s think about the two-slit experiment in terms of the Heisenberg Uncertainty Principle.

Recall that the Uncertainty principle states that there is a small minimum value for the product of the uncertainties of position and momentum.

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

This concept can be used to describe why a light wave is diffracted by a slit. As the photon or other wave-particle passes through the slit, the uncertainty of the position of the wave-particle is basically given by the size of the slit. The uncertainty in momentum then allows for the spreading of the wave-particle spatially. This is illustrated in the diagram. This interpretation is very useful in understanding how Einstein used this experiment as a criticism of the Uncertainty Principle and of the Quantum Theory itself.



In 1924 [8] [9], Louis de Broglie proposed a wave description of all matter by proposing his famous wavelength relationship

$$\lambda = h/p$$

His predictions that matter-wave interference could be observed was confirmed in 1927 in independent experiments by George Thomson, who observed diffraction patterns in electron beams passing through thin metal films [10] and by Clinton Davison and Lester Germer, who observed electron diffraction on an electron beam focused on a crystalline nickel metal surface. [11] Thomson and Davisson shared the Nobel Prize in Physics in 1937 for these discoveries.

While the observation of interference of matter waves gave a great deal of credibility to the emerging quantum theory, Einstein was still troubled. In a series of interactions with Bohr, Einstein would propose thought experiments which he believed would uncover an inconsistency in the quantum theory by violating the Heisenberg Uncertainty Principle. Bohr would then consider the experiment and, in particular, the apparatus that would be used to make the measurements Einstein had proposed. Then, in presenting the "apparatus" to Einstein, Bohr would explain the flaw in Einstein's reasoning and how such a measurement could not violate the predictions of quantum mechanics.

One such exchange occurred over the concept of the "two-slit" experiment. In this experiment, a beam of electrons travels through a screen before arriving at a detector. In the screen, there are two slits through which the beam may pass. Each of these slits will diffract the beam, and lead to an interference pattern as the beam hits a detector screen. The diffraction is confirmed by the interference pattern observed on the detector.

To make matters even more interesting, if one slit is blocked, the result is the disappearance of the interference pattern. Instead, the recorded signal is consistent with the electrons traveling through the single unblocked slit.

For light waves, this phenomenon was well understood, thanks to the experiments of Young. But for matter waves, the picture becomes someone bizarre. There is not much of a problem if one considers what happens when the beam is turned on continuously. In this case, there are plenty of electrons making the transit and it is easy to imagine each as having a wave nature which can interfere with all of the other electrons making the transit.

The real excitement happens when the electron source is slowed down so that only one electron is making the transit at a time. If the resulting signals generated when the electrons reach the detector are integrated, over time an identical interference pattern emerges! "How can that be?" I hear you cry. And the question would indeed be very profound.

One explanation is that each electron traverses the distance from the gun through the slits by taking both possible pathways. This explanation is equivalent to saying that the electron becomes delocalized as soon as it leaves the source, takes all possible pathways to the detector and then becomes localized once again when it interacts with the detector, revealing its final position. Such an explanation would be very problematic to a person clinging to the philosophy of Determinism.

Einstein's description of the phenomenon provided an important piece of the puzzle in terms of probing the limitations of quantum theory. Einstein argued that a particle passing through a slit would only have its path altered if it imparted some momentum to the screen containing the slit through a collision. That collision would have to cause the screen to move a tiny amount (due to conservation of momentum.) And if that movement could be detected, then one would then simultaneously know both the position of the particle (as it passed through the slit) and its momentum (due to the momentum imparted to the slit itself.) And this would create a violation of the Heisenberg Uncertainty principle.

Bohr's response was quick and decisive. He pointed to the fact that Einstein had only attempted to apply the Uncertainty Principle to the wave-particle that passed through the slit and not to the slit itself. In fact, the uncertainty in the momentum of the slit will be the same as the uncertainty in the momentum of the wave-particle (since similar methods are used to measure them.)

$$\Delta p_{slit} = \Delta p_{wp}$$

Further, the uncertainty of the position of the wave-particle is equal to the uncertainty of the position of the of the slit.

$$\Delta x_{slit} = \Delta x_{wp}$$

Additionally, the slit itself must satisfy the Uncertainty Principle in that

$$\Delta x_{slit} \Delta p_{slit} \geq \hbar/2$$

simple substitution shows that if the slit is governed by the Uncertainty Principle, then the wave-particle must be as well.

$$\Delta x_{wp} \Delta p_{wp} \geq \hbar/2$$

This argument does not prove that quantum mechanics is correct, but it does show that it is self-consistent.

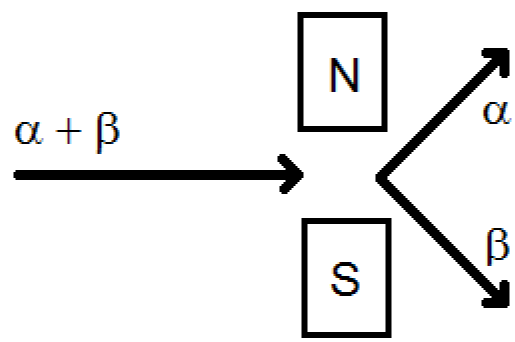
Very recently, scientists have used a modified approach to the double-slit experiment to reopen the question. [12] In this experiment, laser light shines on a screen with two pinholes. A clever detection system is used that detects only those photons that pass through one of the pinholes (a particle-like behavior.) But at the same time, detecting wires are placed in the positions of the destructive interference fringes (where no light should fall), confirming that no light is detected in these dark fringes (which is a consequence of the wave nature of light.) As such, the experiments demonstrate that light can show both the wave and particle nature simultaneously – something that Bohr had predicted to be impossible based on the idea of complementarity. Clearly, the debate continues and forms the subject of current research.

Bohr and Einstein would have several of these types of debates over the course of the late 1920s. Each time, Einstein would propose a thought experiment which he believed would violate the Uncertainty Principle, and each time Bohr would counter with a demonstration that, in fact, there was no violation at all. It seemed that Einstein was defeated. However, that was far from the case!

However, before exploring Einstein's next move, let's consider another experiment that shows the strangeness of quantum mechanics. It will be useful in framing a discussion of Einstein's next move.

The Stern-Gerlach Experiment

One of the very interesting aspects of many small particles, including electrons, is that of spin. (The original Stern-Gerlach experiment [13] was performed on a beam of silver atoms, but the result apply to electrons as well.) The property of spin creates a magnetic moment for these particles. For electrons, which have **s** = ½, the component of angular momentum along an external axis can take two possible values, $m_s = \pm \frac{1}{2}$. That means that an electron traveling through an inhomogeneous magnetic field can align its magnetic moment either with or against the external field. The ramifications are very interesting.



A beam of electrons that passes through an inhomogeneous magnetic field will be split into two beams. Those electrons whose magnetic moment aligned with the field will be deflected in one direction, and those with a magnetic field aligned against the external field will be deflected in the other. Each beam can then be considered as containing only electrons that are either "spin up" (α, $m_s = +\frac{1}{2}$) or "spin down" (β, $m_s = -\frac{1}{2}$). As such, if one of the beams passes through another magnetic field that it oriented parallel to the first, no further splitting occurs since all of the electrons in that sub-beam have their spins aligned.

However, things get very interesting when the second magnetic field is oriented at 90° to the first. Since the magnetic moments of the electrons are aligned perpendicular to the external magnetic field, there should be no effect. What actually happens is that the beam again splits into two sub-beams, just as the original beam did!

If the second magnetic field is placed at some other angle, the beam will still split into two components, but the intensities will be determined by the magnitude of the projection of the electron magnetic moment along the external axis. That magnitude is easily calculable if one thinks of the spin wavefunction as a linear combination of two spin functions in the rotated axis system.

$$\Psi_{spin} = \frac{1}{\sqrt{2}}\cos(\theta)\cdot\alpha + \frac{1}{\sqrt{2}}\sin(\theta)\cdot\beta$$

where θ is the angle between the two magnetic fields. The factors of $\frac{1}{\sqrt{2}}$ are to normalize the wavefunction. The probabilities then of measuring the spin as either an α or β state is given by the squares of the corresponding Fourier coefficients.

$$P(\alpha) = \tfrac{1}{2}\cos^2(\theta)$$
$$P(\beta) = \tfrac{1}{2}\sin^2(\theta)$$

This conclusion will be useful in interpreting later results.

One very important question that the Stern-Gerlach result raises deals directly with Determinacy. The question is whether or not an individual electron “knows” that it is α or β before interacting with the detector. The results (particularly for the experiments where a beam of selected spin particles is resplit) suggests that it is the interaction with the detector that forces the particle into one state or the other.

In this manner, the Stern-Gerlach result shows is that making a measurement on a system will, in fact, alter that system. The interaction of the electrons with the external field causes an alignment of the individual magnetic moments (either with or against the external field.)

The types of experiments (and specifically spin detectors) used in the Stern-Gerlach experiment can be used to help to frame the next step in the Einstein-Bohr debates on the completeness of quantum mechanics.

Spooky Action at a Distance

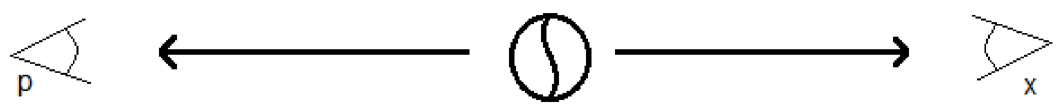
In 1935, Einstein raised the stakes in the quantum debate significantly. Along with his postdoctoral co-authors, Boris Podolsky and Nathan Rosen, published one of the most famous papers in the history of the quantum theory debates. The EPR paper [14] (so called based on the initials of the authors) would create a veritable firestorm within the community that championed the Copenhagen interpretation of Quantum Mechanics.

The EPR paradox

The EPR paper proposed a paradox in the form of a thought experiment, much as the several thought experiments proposed by Einstein to Bohr at the various Solvay Conferences. In the paradox, Einstein used the concepts of a conserved center of mass and conserved momentum in a fragmenting particle to show that either a measurement on one fragment must affect the properties of the other, or that the quantum theory had to be incomplete.

The thought experiment involved the fragmentation of a particle into two fragment particles. The fragment particles would be linked through a single wavefunction describing the entire system. After some time of traveling apart, it was assumed that the two fragment particles could no longer interact as they were physically separated by a distance.

At some point following the fragmentation, the position is measure for one of the fragment particles. This, thought the conservation of the center of mass, would determine the position of the other particle. Then, by measuring the momentum of the counter fragment, the momentum of the first fragment would be determined through the conservation of momentum. As such, there would be simultaneous knowledge of both position and momentum for both particles, in violation of the Uncertainty Principle.



The argument in the EPR paper was that since a measurement on one fragment determined the properties of the counter fragment, and that the two fragments were separated in space, that the properties of the counter fragment must have been determined all along, irrespective of having been measured. (Einstein referred to the phenomenon of measurement on one fragment affecting properties on the counter fragment as "Spooky action at a distance.) In other words, Indeterminacy as suggested by the Heisenberg Uncertainty Principle must be a fallacy. The only other explanation possible was that the Quantum Theory had to be incomplete. With this argument, people had to take very seriously the possibility that a theory of "local reality" in which properties exist with definite values, as opposed to only coming to being through the interaction with a detector of some sort, as a distinct possibility.

Bohr responded within months. He attacked a specific assumption of the set up of the EPR paradox, namely that a measurement of the properties of one particle would not "disturb the system in any way."

Hidden Variables

The EPR paradox was both eloquent and succinct. It touched off quite a storm within the community as well as it shock the very foundations of the quantum theory. But perhaps even more interestingly, it spurned a whole new avenue of research into understanding the ramifications. Specifically troubling was the idea that the wavefunction describing a system did not, in fact, provide a complete description of that system.

Scientists began to wonder if there might be some "hidden variables" in a system that allowed properties to be both hidden under the vagueness of a wavefunction and also determined by the definite values of the variables, irrespective of whether or not the system was observed or measured.

In 1951, David Bohm published a textbook [15] on quantum theory that included a good deal of discussion on the EPR paradox. In it, he suggested measuring the nuclear spins of hydrogen atoms that result from the dissociation of a singlet-state hydrogen molecule. The spins would be correlated through the conservation of angular momentum and could thus take the place of the measurements of position and momentum in the EPR version.

In Bohm's version of the EPR experiment (sometimes called the EPRB experiment) the spin states of the hydrogen atoms would be correlated as the atoms would be "entangled". And since angular momentum had to be conserved, measurement of the spin of one atom along the laboratory fixed z-axis would determine the value along the z-axis for the other atom. But what if the measurement was made along the x- or y-axes? If the EPR definition of reality is to be believed, these values must also be determined (or real.) Of course quantum mechanics only allows for the measurement of one of the components, as the operators for the three components do not commute. Thus, if the EPR definition of reality is correct, then the wavefunction by necessity must be incomplete. There would need to be hidden variables.

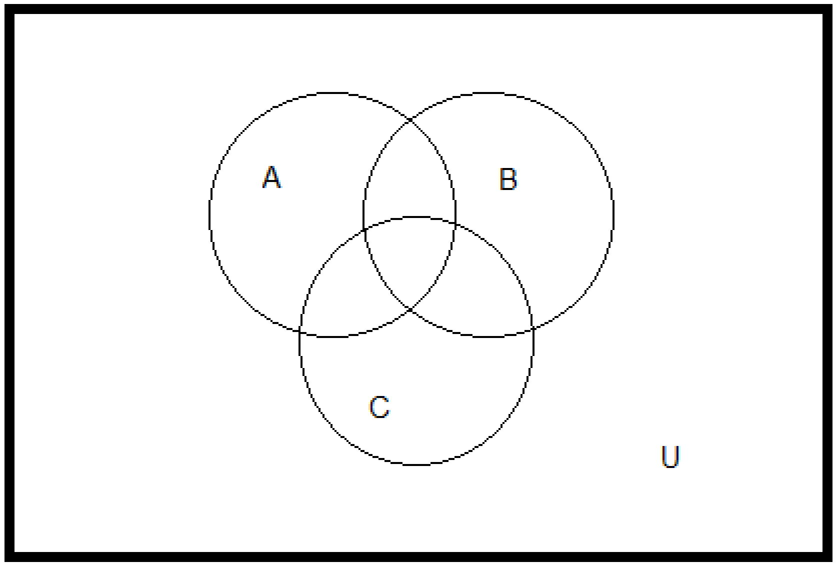
Even more significantly, Bohm's proposed experiments could be carried out in a laboratory, rather than being limited to the realms of thought.

Bell's Inequality

Bohm's work on the EPR paradox reawakened an interest in the topic. One physicist who took a particular interest in the topic was John S. Bell. Bell proposed a mathematical model that could in fact distinguish between local hidden variable theories and quantum theory [16].

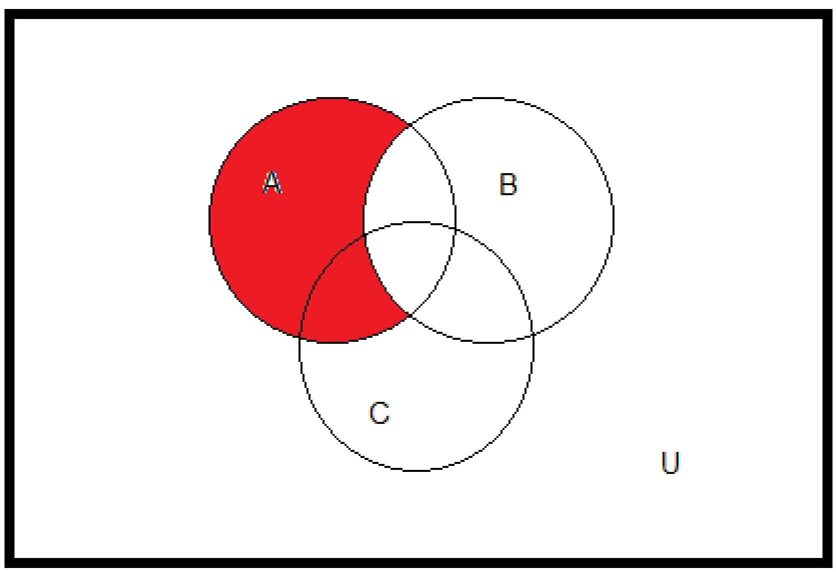
Consider a set of things U which can be subdivided into three overlapping subsets, A, B and C. Bell's theorem states: the number of members of A that are not a member of B plus all members of B that are not a member of C must be greater than or equal to the number of members in the subset of A that are not also in subset B.

To show this, let's first settle on some notation. We'll call the number of items that are in subset A, but not in subset B by the symbol $N(A_+B_-)$ and the number of items in subset B but not in subset C by $N(B_+C_-)$. Etcetera. This notation coupled with the use of some Venn diagrams, the concept of the inequality should become clear.



It should be clear that $N(A_+B_-)$ can be easily shown to be given by the number of items in subset A, not in subset B and in subset C, plus the number in A, not in B and not in C.

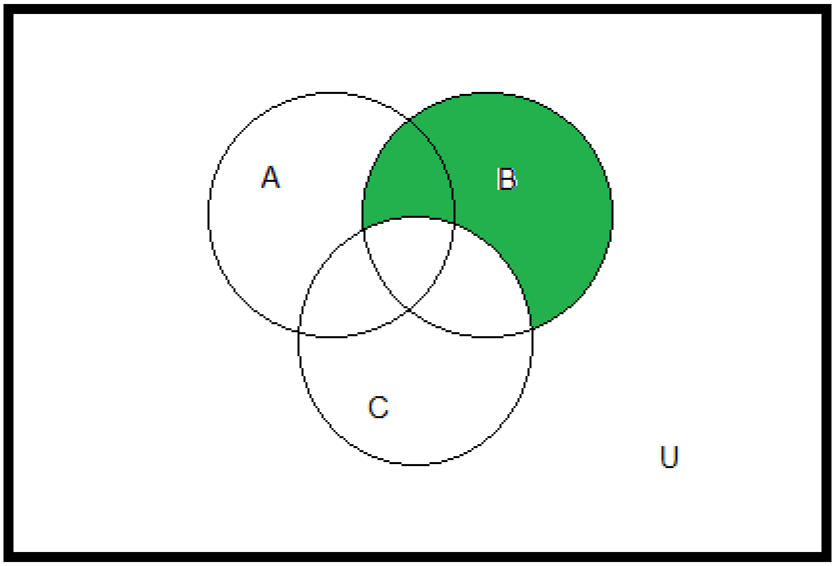
$$N(A_+B_-) = N(A_+B_-C_+) + N(A_+B_-C_-)$$



Similar sums can be derived for $N(B_+C-)$ and $N(A_+C_-)$

$$N(B_+C_-) = N(A_+B_+C_-) + N(A_-B_+C_-)$$
$$N(A_+C_-) = N(A_+B_+C_-) + N(A_+B_-C_-)$$

Shown below is the sum for $N(B_+C_-)$.



Adding the terms for $N(A_+B_-)$ and $N(B_+C_-)$ gives

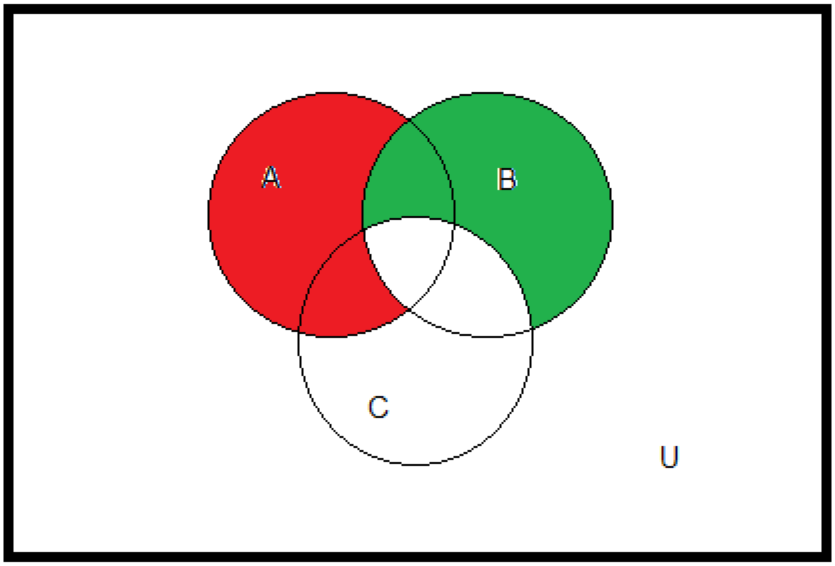
$$N(A_+B_-) + N(B_+C_-) = N(A_+B_-C_+) + N(A_+B_-C_-) + N(A_+B_+C_-) + N(A_-B_+C_-)$$

This can be simplified by grouping the terms for $N(A_+B_+C_-)$ and $N(A_+B_-C_-)$ and recognizing that their sum gives $N(A_+C_-)$.

$$N(A_+B_-) + N(B_+C_-) = N(A_+B_-C_+) + N(A_-B_+C_-) + N(A_+C_-)$$

So long as neither $N(A_+B_-C_+)$ nor $N(A_-B_+C_-)$ are negative (which they can not be) then we arrive at Bell's inequality:

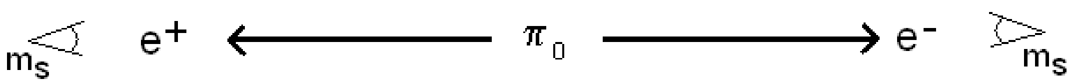
$$N(A_+B_-) + N(B_+C_-) \geq N(A_+C_-)$$



Employing the Stern-Gerlach results to Test Bell's Inequality

On the face of it, Bell's result does not seem that extraordinary. In fact, it almost seems trivial. However, it is only trivial when the results of tests that would place an object into group A, B or C are not correlated. When the results are correlated, the result becomes a bit perplexing.

Consider the dissociation of a pion (also called a π meson), which is a subatomic particle with zero spin and zero charge. It can decompose into a positron and an electron (to conserve charge), each traveling in opposite directions (such that momentum is conserved.) The spins will also be entangled in such a way as to conserve angular momentum.



In fact, the spin state of the electron/positron pair will be given by the familiar singlet spin function:

$$\Psi = \frac{1}{\sqrt{2}}\left(\alpha_+\beta_- - \beta_+\alpha_-\right)$$

This suggests that if the positron (subscript +) is detected in the α spin state, the electron (subscript -) will necessarily be forced into the β spin state. The wavefunction allows for equal probability that the positron will be detected in the α spin state or the β spin state, but detection in either state forces an immediate collapse of the wavefunction for the electron. This is the "spooky action at a distance" that Einstein so vehemently rejected in the EPR paper [14]. Einstein also insisted that the spin state of the positron was a "real" property that existed with a definite value for the entire transit of the positron from the decay event to the detector. And quantum mechanics, in Einstein's view, was incomplete in that it could not predict the "realness" of that spin state. If Einstein's view was correct, then correlated measurements of the two spin states would have to satisfy Bell's inequality.

With the results of the Stern-Gerlach experiments, we can actually determine exactly what quantum mechanics will predict. To do this, we will set up our detectors to detect the spin to the dissociated fragments, but we will rotate the detectors relative to one another. In a laboratory-fixed coordinate system, we will set detector A at 0° rotation, B at 30° and C and 60°. What we want to know is the probability that if one detector measures its particle to be in spin state α that the other will measure its particle to be in spin state β. That probability will be related to the angle of rotation of the second detector relative to the first. According to the Stern-Gerlach result, the probability is given by ½ $\sin(\theta_2-\theta_1)$, where θ_2 and θ_1 are the angles of the second and first detectors in the pair respectively.

So if we define $P(A_+B_-)$ as the probability that detector A detects an α spin and detector B fails to detect a β spin, we can construct the following table based on three specific experimental configurations:

Experiment	θ_1	θ_2	Case	$\theta_2 - \theta_1$	½ $\sin^2(\Delta\theta)$
1	0°	30°	$P(A_+B_-)$	30°	0.125
2	30°	60°	$P(B_+C_-)$	30°	0.125
3	0°	60°	$P(A_+C_-)$	60°	0.375

After collecting data from a very large set of measurements using these configurations, we will have can compare the experimental distribution of outcomes to what is predicted by quantum mechanics, and thus conclude if it is possible to have a locality variable that predetermines our outcomes, or if the measurements are purely probabilistic. If the locality variable exists, then Bell's Inequality must hold [17].

$$P(A_+B_-) + P(B_+C_-) \geq P(A_+C_-)$$

However, if Quantum Mechanics allows for a locality variable to redetermine the measured outcomes of the three experiments, then the following must be true:

$$0.125 + 0.125 \geq 0.375$$

Except that it simply isn't true. (In fact, it isn't even true for extremely large values of the sum 0.125 + 0.125.) The above set of experiments was proposed by Alain Aspect in 1976 [17], and results published in 1982 [18]. And while the results were criticized due to the "detection loophole", results of similar experiments being conducted up to 2015 [20] confirmed Aspect's results. Alain Aspect shared the 2022 Nobel Prize in Physics with John Clauser and Anton Zeillinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science". [21]

Since Aspect's result was derived completely independent of any theory of hidden variables, it should be clear that the result is incompatible with any such theory. In fact, the result shows that one must divorce oneself from any ideas of local realism for quantum mechanical particles. One simply must conclude that it is the observation that creates the reality and that no reality for observable properties on quantum mechanical system can exist independent of their observation. (Of course, Sheldon Cooper would also point out that one can be beaten up simply for referring to oneself as "one.) [19]

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