Appendix II – Selected Character Tables

Nonaxial Groups

C_1	E
A	1

C_s	E	σ		
A’	1	1	x, y, R_z	x^2, y^2, z^2, xy
A”	1	-1	z, R_x, R_y	xz, yz

C_i	E	i		
A_g	1	1	R_x, R_y, R_z	x^2, y^2, z^2, xy, xz, yz
A_u	1	-1	x, y, z	

C_n groups

C_2	E	C_2		
A	1	1	z, R_z	x^2, y^2, z^2, xy
B	1	-1	x, y, R_x, R_y	xz, yz

C_3	E	C_3	C_3^2		
A	1	1	1	z, R_z	$x^2 + y^2$, z^2
E	1	ε	ε^*	x+iy; R_x+iR_y	(x^2-y^2, xy)
	1	ε^*	ε	x-iy; R_x-iR_y	(xz, yz)

C_4	E	C_4	C_2	C_4^3		
A	1	1	1	1	z, R_z	$x^2 + y^2$, z^2
B	1	-1	1	-1		x^2-y^2, xy
E	1	i	-1	$-i$	x+iy; R_x+iR_y	(xz, yz)
	1	$-i$	-1	i	x-iy; R_x-iR_y	

C_5	E	C_5	C_5^2	C_5^3	C_5^4		
A	1	1	1	1	1	z, R_z	$x^2 + y^2$, z^2
E_1	1	ε	ε^2	ε^{2*}	ε^*	x+iy, R_x+iR_y	(xz, yz)
	1	ε^*	ε^{2*}	ε^2	ε	x-iy, R_x-iR_y	

E_2	$\begin{cases} 1 & \varepsilon^2 & \varepsilon^* & \varepsilon & \varepsilon^{2*} \\ 1 & \varepsilon^{2*} & \varepsilon & \varepsilon^* & \varepsilon^2 \end{cases}$	$(x^2\text{-}y^2, xy)$

C_6	E	C_6	C_6^2	C_6^3	C_6^4	C_6^5		
A	1	1	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
B	1	-1	1	-1	1	-1		
E_1	$\begin{cases} 1 \\ 1 \end{cases}$	ε ε^*	$-\varepsilon^*$ $-\varepsilon$	-1 -1	$-\varepsilon$ $-\varepsilon^*$	ε^* ε	x+iy, R_x+iR_y x-iy, R_x-iR_y	(xz, yz)
E_2	$\begin{cases} 1 \\ 1 \end{cases}$	$-\varepsilon^*$ $-\varepsilon$	$-\varepsilon$ $-\varepsilon^*$	1 1	$-\varepsilon^*$ $-\varepsilon$	$-\varepsilon$ $-\varepsilon^*$		$(x^2\text{-}y^2, xy)$

D_n groups

D_2	E	$C_2(z)$	$C_2(y)$	$C_2(x)$			
A_1	1	1	1	1			X^2, y^2, z^2
B_1	1	1	-1	-1	z	R_z	xy
B_2	1	-1	1	-1	y	R_y	xz
B_3	1	-1	-1	1	x	R_x	yz

D_3	E	2 C_2	3 C_2'		
A_1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	-1	z, R_z	
E	2	-1	0	(x, y) (R_x, R_y)	$(x^2\text{-}y^2, xy)$ (xz, yz)

D_4	E	2 C_4	C_2	2 C_2’	2 C_2”		
A_1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	-1	-1	z, R_z	
B_1	1	-1	1	1	-1		$x^2\text{-}y^2$
B_2	1	-1	1	-1	1		xy
E	2	0	-2	0	0	(x, y) (R_x, R_y)	(xz, yz)

D_5	E	2 C_5	2 C_5’	5 C_2		
A_1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	-1	z R_z	
E_1	2	2 cos(72°)	2 cos(144°)	0	(x, y) (R_x, R_y)	(xz, yz)
E_2	2	2 cos(144°)	2 cos(72°)	0		$(x^2\text{-}y^2, xy)$

D_6	E	2 C_6	2 C_3	C_2	3 C_2'	3 C_2''		
A_1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	-1	-1	z, R_z	
B_1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1		
E_1	2	-1	1	-2	0	0	(x, y) (R_x, R_y)	(xz, yz)
E_2	2	-1	-1	2	0	0		(x^2-y^2, xy)

C_{nv} groups

C_{2v}	E	C_2	σ_v	σ_v'			
A_1	1	1	1	1	z		x^2, y^2, z^2
A_2	1	1	-1	-1		R_z	xy
B_1	1	-1	1	-1	x	R_y	xz
B_2	1	-1	-1	1	y	R_x	yz

C_{3v}	E	2 C_2	3 σ_v		
A_1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	-1	R_z	
E	2	-1	0	(x, y) (R_x, R_y)	(x^2-y^2, xy) (xz, yz)

C_{4v}	E	2 C_4	C_2	2 σ_v	2 σ_d		
A_1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	-1	-1	R_z	
B_1	1	-1	1	1	-1		x^2-y^2
B_2	1	-1	1	-1	1		xy
E	2	0	-2	0	0	(x, y) (R_x, R_y)	(xz, yz)

C_{5v}	E	2 C_5	C_5^2	5 σ_v		
A_1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	-1	R_z	
E_1	2	2 cos(72°)	2 cos(144°)	0		(xz, yz)
E_2	2	2 cos(144°)	2 cos(72°)	0	(x, y) (R_x, R_y)	(x^2-y^2, xy)

C_{6v}	E	2 C_6	2 C_3	C_2	3 σ_v	3 σ_d		
A_1	1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	1	-1	-1	R_z	

B_1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1		
E_1	2	1	-1	-2	0	0	(x, y) (R_x, R_y)	(xz, yz)
E_2	2	-1	-1	2	0	0		(x^2-y^2, xy)

C_{nh} Groups

C_{2h}	E	C_2	i	σ_h		
A_g	1	1	1	1	R_z	X^2, y^2, z^2
A_u	1	1	-1	-1	z	
B_g	1	-1	1	-1	R_x, R_y	xz, xy, yz
B_u	1	-1	-1	1	x, y	

C_{3h}	E	C_3	C_3^2	σ_h	S_3	S_s^2		
A’	1	1	1	1	1	1	R_z	$x^2 + y^2$, z^2
E’	1	ε	ε^*	1	ε	ε^*	x+iy	(x^2-y^2, xy)
	1	ε^*	ε	1	ε^*	ε	x-iy	
A”	1	1	1	-1	-1	-1	z	
E”	1	ε	ε^*	-1	$-\varepsilon$	$-\varepsilon^*$	R_x+iR_y	(xz, yz)
	1	ε^*	ε	-1	$-\varepsilon^*$	$-\varepsilon$	R_x-iR_y	

C_{4h}	E	C_4	C_2	C_4^3	i	S_4^3	σ_h	S_4		
A_g	1	1	1	1	1	1	1	1	R_z	$x^2 + y^2$, z^2
B_g	1	-1	1	-1	1	-1	1	-1		x^2-y^2, xy
E_g	1	i	-1	$-i$	1	i	-1	$-i$	R_x+iR_y	(xz, yz)
	1	$-i$	-1	i	1	$-i$	-1	i	R_x-iR_y	
A_u	1	1	1	1	-1	-1	-1	-1	z	
B_u	1	-1	1	-1	-1	1	-1	1		
E_u	1	i	-1	$-i$	-1	i	1	$-i$	x+iy	
	1	$-i$	-1	i	-1	$-i$	1	i	x-iy	

C_{5h}	E	C_5	C_5^2	C_5^3	C_5^4	σ_h	S_5	S_5^2	S_5^3	S_5^4		
A’	1	1	1	1	1	1	1	1	1	1	R_z	$x^2 + y^2$, z^2
E_1’	1	ε	ε^2	ε^{2*}	ε^*	1	ε	ε^2	ε^{2*}	ε^*	x+iy	
	1	ε^*	ε^{2*}	ε^2	ε	1	ε^*	ε^{2*}	ε^2	ε	x-iy	

E_2'	$\begin{cases}1 & \varepsilon^2 & \varepsilon^* & \varepsilon & \varepsilon^{2*} & 1 & \varepsilon^2 & \varepsilon^* & \varepsilon & \varepsilon^{2*} \\ 1 & \varepsilon^{2*} & \varepsilon & \varepsilon^* & \varepsilon^2 & 1 & \varepsilon^{2*} & \varepsilon & \varepsilon^* & \varepsilon^2 \end{cases}$										$(x^2-y^2,$ xy)
A"	1	1	1	1	1	-1	-1	-1	-1	-1	z	
E_1"	$\begin{cases}1 & \varepsilon & \varepsilon^2 & \varepsilon^{2*} & \varepsilon^* & -1 & -\varepsilon & -\varepsilon^2 & -\varepsilon^{2*} & -\varepsilon^* \\ 1 & \varepsilon^* & \varepsilon^{2*} & \varepsilon^2 & \varepsilon & -1 & -\varepsilon^* & -\varepsilon^{2*} & -\varepsilon^2 & -\varepsilon \end{cases}$										R_x+iR_y R_x-iR_y	(xz, yz)
E_2"	$\begin{cases}1 & \varepsilon^2 & \varepsilon^* & \varepsilon & \varepsilon^{2*} & -1 & -\varepsilon^2 & -\varepsilon^{2*} & -\varepsilon & -\varepsilon^{2*} \\ 1 & \varepsilon^{2*} & \varepsilon & \varepsilon^* & \varepsilon^2 & -1 & -\varepsilon^{2*} & -\varepsilon^2 & -\varepsilon^* & -\varepsilon^2 \end{cases}$											

D_{nh} Groups

D_{2h}	E	$C_2(z)$	$C_2(y)$	$C_2(x)$	i	σ_{xy}	σ_{xx}	σ_{yz}		
A_g	1	1	1	1	1	1	1	1		x^2, y^2, z^2
B_{1g}	1	1	-1	-1	1	1	-1	-1	R_z	xy
B_{2g}	1	-1	1	-1	1	-1	1	-1	R_y	xz
B_{3g}	1	-1	-1	1	1	-1	-1	1	R_x	yz
A_u	1	1	1	1	-1	-1	-1	-1		
B_{1u}	1	1	-1	-1	-1	-1	1	1	z	
B_{2u}	1	-1	1	-1	-1	1	-1	1	y	
B_{3u}	1	-1	-1	1	-1	1	1	-1	x	

D_{3h}	E	2 C_3	3 C_2'	σ_h	2 S_3	3 σ_v		
A_1'	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2'	1	1	-1	1	1	-1	R_z	
E'	2	-1	0	2	-1	0	(R_x, R_y)	(x^2-y^2, xy)
A_1"	1	1	1	-1	-1	-1		
A_2"	1	1	-1	-1	-1	1	z	
E"	2	-1	0	-2	1	0	(x, y)	(xz, yz)

D_{4h}	E	2 C_4	C_2	2 C_2'	2 C_2"	i	2 S_4	σ_h	2 σ_v	2 σ_d		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	R_z	
B_{1g}	1	-1	1	1	-1	1	-1	1	1	-1		x^2-y^2
B_{2g}	1	-1	1	-1	1	1	-1	1	-1	1		xy
E_g	2	0	-2	0	0	2	0	-2	0	0	(R_x, R_y)	(xz, yz)
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1		
A_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	z	
B_{1u}	1	-1	1	1	-1	-1	1	-1	-1	1		
B_{2u}	1	-1	1	-1	1	-1	1	-1	1	-1		
E_u	2	0	-2	0	0	-2	0	2	0	0	(x, y)	

D_{6h}	E	2 C_6	2 C_3	C_2	3 C_2'	3 C_2"	i	2 S_3	2 S_6	σ_h	3 σ_v	3 σ_d		
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$

A_{2g}	1	1	1	1	-1	-1	1	1	1	1	-1	-1	R_z	
B_{1g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
B_{2g}	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1		
E_{1g}	2	-1	1	-2	0	0	2	-1	1	-2	0	0	(R_x, R_y)	(xz, yz)
E_{2g}	2	-1	-1	2	0	0	2	-1	-1	2	0	0		(x^2-y^2, xy)
A_{1u}	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1		
A_{2u}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	z	
B_{1u}	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		
B_{2u}	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		
E_{1u}	2	-1	1	-2	0	0	-2	-1	1	2	0	0	(x, y)	
E_{2u}	2	-1	-1	2	0	0	-2	1	1	-2	0	0		

D_{nd} Groups

D_{2d}	E	2 S_4	C_2	2 C_2'	2 σ_d		
A_1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	-1	-1	R_z	
B_1	1	-1	1	1	-1		x^2-y^2
B_2	1	-1	1	-1	1	z	xy
E	2	0	-2	0	0	(x, y) (R_x, R_y)	(xz, yz)

D_{3d}	E	2 C_3	3 C_2'	i	2 S_6	3 σ_d		
A_{1g}	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	-1	1	1	-1	R_z	
E_g	2	-1	0	2	-1	0	(R_x, R_y)	$(x^2-y^2, xy), (xz, yz)$
A_{1u}	1	1	1	-1	-1	-1		
A_{2u}	1	1	-1	-1	-1	1	z	
E_u	2	-1	0	-2	1	0	(x, y)	

D_{4d}	E	2 S_8	2 C_4	2 S_8^3	C_2	4 C_2'	4 σ_d		
A_1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	1	-1	-1	R_z	
B_1	1	-1	1	-1	1	1	-1		
B_2	1	-1	1	-1	1	-1	1	z	
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	0	(x, y)	
E_2	2	0	-2	0	2	0	0		(x^2-y^2, xy)
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	0	(R_x,R_y)	(xz, yz)

S_n Groups

Cubic Groups

T_d	E	8 C_3	3 C_2	6 S_4	6 σ_d		
A_1	1	1	1	1	1		$x^2+y^2+z^2$
A_2	1	1	1	-1	-1		
E	2	-1	2	0	0		$(2z^2-x^2-y^2, x^2-y^2)$
T_1	3	0	-1	1	-1	(R_x, R_y, R_z)	
T_2	3	0	-1	-1	1	(x, y, z)	(xy,xz,yz)

O_h	E	$8C_3$	$6C_2$	$6C_4$	$3C_2$	i	$6S_4$	$8S_6$	$3\sigma_h$	$6\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$x^2+y^2+z^2$
A_{2g}	1	1	-1	-1	1	1	-1	1	1	-1		
E_g	2	-1	0	0	2	2	0	-1	-2	0		$(2z^2-x^2-y^2, x^2-y^2)$
T_{1g}	3	0	-1	1	-1	3	1	0	-1	-1	(R_x, R_y, R_z)	
T_{2g}	3	0	1	-1	-1	3	-1	0	-1	1		(xy,xz,yz)
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1		
A_{2u}	1	1	-1	-1	1	-1	1	-1	-1	1		
E_u	2	-1	0	0	2	-2	0	1	-2	0		
T_{1u}	3	0	-1	1	-1	-3	-1	0	1	1	(x, y, z)	
T_{2u}	3	0	1	-1	-1	-3	1	0	1	-1		

Icosahedral Group

Linear Groups