

Appendix I

Some Useful Mathematical Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^{n+1} \frac{x^{2n-1}}{(2n-1)!}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!}$$

$$e^{\pm i\theta} = \cos(\theta) \pm i \sin(\theta) \qquad \cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2} \qquad \sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2}$$

Some Useful Integrals

$$\int \sin(\alpha x) \sin(\beta x) dx = \frac{\sin[(\alpha - \beta)x]}{2(\alpha - \beta)} - \frac{\sin[(\alpha + \beta)x]}{2(\alpha + \beta)} \quad \alpha \neq \beta$$

$$\int \sin^2(\alpha x) dx = \frac{x}{2} - \frac{\sin(2\alpha x)}{4\alpha} \qquad \int \sin^3(\alpha x) dx = \frac{\cos(3\alpha x) - 9\cos(\alpha x)}{12\alpha}$$

$$\int x \sin^2(\alpha x) dx = \frac{x^2}{4} - \frac{x \sin(2\alpha x)}{4\alpha} - \frac{\cos(2\alpha x)}{8\alpha^2}$$

$$\int x^2 \sin^2(\alpha x) dx = \frac{x^3}{6} - \left(\frac{x^2}{4\alpha} - \frac{1}{8\alpha^3} \right) \sin(2\alpha x) - \frac{x \cos(2\alpha x)}{4\alpha^2}$$

$$\int \cos(\alpha x) \cos(\beta x) dx = \frac{\sin[(\alpha - \beta)x]}{2(\alpha - \beta)} + \frac{\sin[(\alpha + \beta)x]}{2(\alpha + \beta)}$$

$$\int \cos^2(\alpha x) dx = \frac{x}{2} + \frac{1}{4\alpha} \sin(2\alpha x) \qquad \int x \cos^2(\alpha x) dx = \frac{x^2}{4} + \frac{x \sin(2\alpha x)}{4\alpha} + \frac{\cos(2\alpha x)}{8\alpha^2}$$

$$\int x^2 \cos^2(\alpha x) dx = \frac{x^3}{6} + \left(\frac{x^2}{4\alpha} - \frac{1}{8\alpha^3} \right) \sin(2\alpha x) + \frac{x \cos(2\alpha x)}{4\alpha^2}$$

$$\int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}} \quad (n \text{ a positive integer})$$

$$\int_0^\infty e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{4\alpha}}$$

$$\int_0^\infty x^{2n} e^{-\alpha x^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} \alpha^n} \sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^\infty x^{2n+1} e^{-\alpha x^2} dx = \frac{n!}{2\alpha^{n+1}}$$

Some Useful Coordinate Transformations

Plane Polar Coordinates: $0 \leq r \leq \infty; \quad 0 \leq \theta \leq 2\pi$

$$x = r \cos \theta \quad y = r \sin \theta \quad \theta = \arctan\left(\frac{y}{x}\right) \quad r = \sqrt{x^2 + y^2}$$

$$\begin{aligned} \nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \\ &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \end{aligned}$$

Spherical Polar Coordinates: $0 \leq r \leq \infty; \quad 0 \leq \theta \leq \pi; \quad 0 \leq \phi \leq 2\pi$

$$x = r \sin \theta \cos \phi \quad y = r \sin \theta \sin \phi \quad z = r \cos \theta$$

$$\begin{aligned} \nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \\ &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \end{aligned}$$

$$dx \, dy \, dz = r^2 \sin \theta \, dr \, d\theta \, d\phi$$